Dynamical Interpretation of Chemical Freeze-Out Parameters.

V.D. Toneev[1,2], J. Cleymans[3], E.G. Nikonov[1,2], K. Redlich[1,4],

and

A.A. Shanenko [2]

[1] *Gesellschaft für Schwerionenforshung, D-64220 Darmstadt, Germany*

[2] *Bogoliubov Laboratory of Theoretical Physics,*

JINR, 141980 Dubna, Moscow Region, Russia

[3] *Department of Physics, University of Cape Town, Rondebosch 7701, South Africa*

[4] *Institute of Theoretical Physics, University of Wroclaw, PL-64291 Wroclaw, Poland.*

Abstract

It is shown that the condition for chemical freeze-out, average energy per hadron ≈ 1 GeV, selects the softest point of the equation of state, namely the point where the pressure divided by the energy density, $p(\varepsilon)/\varepsilon$ has a minimum. The sensitivity to the equation of state used is discussed. The previously proposed mixed phase model, which is consistent with lattice QCD data naturally leads to the chemical freeze-out condition.

Over the last few years the question of chemical equilibrium in heavy ion collisions has attracted much attention [1]. Assuming thermal and chemical equilibrium within a statistical model, it has now been shown that it is indeed possible to describe the hadronic abundances produced using beam energies ranging from 1 to 200 AGeV. The observation was made that the parameters of the chemical freeze-out curve obtained at CERN/SPS, BNL/AGS and GSI/SIS all lie on a unique freeze-out curve in the $T - \mu_B$ plane. Recently, a surprisingly simple interpretation of this curve has been proposed: the hadronic composition at the final state is determined solely by an energy per hadron of approximately 1 GeV per hadron in the rest frame of the system under consideration [2, 3]. In this letter we propose a dynamical interpretation of the chemical freeze-out curve and show that it is intimately related to the softest point of equation of state defined by the minimum of the ratio $\frac{p}{\varepsilon}(\varepsilon)$ as a function of ε [4]. Our considerations are essentially based on the recently proposed [5, 6] mixed phase model which is consistent with the available QCD lattice data [7]. The underlying assumption of the Mixed Phase (MP) model is that unbound quarks and gluons *may coexist* with hadrons forming an *homogeneous* quark/gluon–hadron phase [5, 6]. Since the mean distance between hadrons and quarks/gluons in this mixed phase may be of same order as that between hadrons, their interaction with unbound quarks/gluons plays an important role defining the order of the phase transition.

Within the MP model [5, 6] the effective Hamiltonian is written in the quasi particle approximation with the density-dependent mean–field interaction. Under quite general requirements of confinement for color charges, the mean–field potential of quarks and gluons is approximated by the following form:

$$U_q(\rho) = U_g(\rho) = \frac{A}{\rho^\gamma} \tag{1}$$

with *the total density of quarks and gluons*

$$\rho = \rho_q + \rho_g + \sum_j n_j \rho_j$$

where ρ_q and ρ_g are the densities of unbound quarks and gluons outside of hadrons, while ρ_j is the density and n_j is the number of valence quarks inside the hadron of type j. The presence of the total density ρ in (1) corresponds to the inclusion of the interaction between all components of the mixed phase. The approximation (1) recovers two important limiting cases of the QCD interaction, namely, if $\rho \to 0$ the interaction

potential goes to infinity, i.e. an infinite energy should be spent to create an isolated quark or gluon which ensures the confinement of color objects and, in the other extreme case of high energy density corresponding to $\rho \rightarrow \infty$ we obtain the asymptotic freedom regime.

The use of a density-dependent potential (1) for quarks and a hadronic potential described by a modified non-linear mean–field model [8] requires certain constraints, related to thermodynamic consistency, to be fulfilled [5, 6]. For the chosen form of the Hamiltonian these conditions require that $U_g(\rho)$ and $U_q(\rho)$ should be independent of the temperature. From these conditions one also obtains an expression for the form of the quark–hadron potential [5].

A detailed study of the pure gluonic $SU(3)$ case with a first order phase transition allows one to fix the values of the parameters as $\gamma = 0.62$ and $A^{1/(3\gamma+1)} = 250\, MeV$. These values are then generalized to the the $SU(3)$ system including quarks. For the case of quarks of two light flavors at zero baryon density, $n_B = 0$, the MP model is consistent with the results from lattice QCD [7] with a deconfinement temperature $T_{dec} = 153\ MeV$ and the crossover type of the deconfinement phase transition. The model can be extended to baryon-rich systems in a parameter–free way [5].

A particular consequence of the MP model is that for $n_B = 0$ the 'softest point' of the equation of state, as defined in [4], is located at comparatively low values of the energy density: $\varepsilon_{SP} \approx 0.45\ GeV/fm^3$. This value of ε is close to the energy density inside a nucleon and, thus, reaching this value signals us that we are dealing with a single 'big' hadron consisting of deconfined matter. For baryonic matter the softest point is gradually washed out at $n_B \gtrsim 0.4\ n_0$. As shown in [5, 6], this behavior differs drastically from both the interacting hadron gas model which has no soft point and the two–phase approach, based on the bag model, having a first order phase transition by construction and the softest point at $\varepsilon_{SP} > 1\ GeV/fm^3$ independent of n_B [4]. These differences should manifest themselves in the expansion dynamics.

In Fig.1 we show trajectories of the evolution of central Au+Au collisions in the $T-\mu_B$ plane together with the freeze-out parameters obtained from hadronic abundances. The initial state was estimated using a transport model starting from a cylinder in the center-of-mass frame with radius $R = 4\ fm$ and length $L = 2R/\gamma_{c.m.}$ as described in [5, 6]. The subsequent isoentropic expansion was calculated using a scaled hydrodynamical model with the MP equation of state. As seen from the figure, the turning points of these

trajectories correlate nicely with the extracted freeze-out parameters, as was noted in [6], as well as with the smooth curve corresponding to a fixed energy per hadron in the hadronic gas model [2].

The observed correlation is further elucidated in Fig.2. The quantity p/ε is closely related to the square of the velocity of sound and characterizes the expansion speed*, so the system lives for the longest time around the softest point which allows it to reach chemical equilibrium for the strongly interacting components. It is also seen that the position of the softest point correlates with the average energy per hadron being about 1 GeV in all nuclear cases and even for $p\bar{p}$ collision. One should note that the quantity ε/ρ_{had}, where ε is the total energy density, coincides with $\langle E_{had}\rangle / \langle N_{had}\rangle$ considered in [2] only in the case when there are no unbound quarks/gluons in the system. In the MP model, all components are interacting with each other and therefore the quantity $\langle E_{had}\rangle$ is not defined. The admixture of unbound quarks at the softest point ε_{SP} amounts to about 13% and 8% at beam energies $E_{lab} = 150$ and $10\ AGeV$, correspondingly.

The MP equation of state plays a decisive role for the regularity considered here, describing both the order of the phase transition and the deconfinement temperature. The two-phase (bag) model exhibits a first order phase transition with $T_{dec} = 160\ MeV$ and has a *spatially separated* Gibbs mixed phase but the corresponding trajectories in the $T - \mu_B$ plane are quite different from those in the MP model as shown in [6]. The exit point from the Gibbs mixed phase at $E_{lab} = 150$ AGeV is close to the corresponding freeze-out point in Fig.1. However the large differences noted above in ε_{SP} and in its dependence on n_B, mainly caused by the different type of the predicted phase transition, does not lead to the observed correlation with the softest point position in the whole energy range considered. The interacting hadron gas model has no softest point effect as was demonstrated in [5, 6]. This fact is seen also from Fig.2 where at $E_{lab} = 2\ AGeV$ the quark admixture is practically degenerated ($\approx 1\%$) and instead of a minimum there occurs a monotonic fall-off specific for hadronic models with a small irregularity in p/ε near the point $\varepsilon/\rho_{had} = 1\ GeV$ †.

It is noteworthy that similarly to the results presented in Fig.2 the softest point of the equation of state correlates with an average energy per quark, $\varepsilon/\rho \approx 350\ MeV$

*In simple hydrodynamic models, for example, the transverse expansion of a cylindrical source, the evolution is governed by the pressure-to-enthalpy ratio, $p/(p+\varepsilon)$ [9, 10].

†Note that at the SIS energies the chemical freeze-out point practically coincides with the thermal freeze-out [11]

which is close to the constituent quark mass. So, at higher values of ε/ρ we are dealing with a strongly-interacting mixture of highly-excited hadrons and unbound massive quarks/gluons forming (in accordance with Landau's idea [12]) an 'amorphous' fluid suitable for hydrodynamic treatment. Below the soft point the interaction deceases, the relative fraction of unbound quarks/gluons decreases, higher hadronic resonances decay into baryons and light mesons and thereby the value of ε/ρ goes down.

In summary, the unified description of the chemical freeze-out parameters found in [2] is naturally related to the fact that the proposed condition $\langle E_{had}\rangle / \langle N_{had}\rangle \approx 1\ GeV$ selects the softest point of the equation of state where the strongly interacting system stays for a long time. Such a clear correlation is observed for the equation of state of the mixed phase model but not in purely hadronic nor in two–phase models. In this respect the success of the MP model in the dynamical interpretation of the freeze-out regularity may be considered as an argument in favor of a crossover type of the deconfinement phase transition in $SU(3)$ system with massive quarks.

We thank B. Friman, Yu. Ivanov and W. Nörenberg for useful discussions. E.G.N. and V.D.T. gratefully acknowledge the hospitality at the Theory Group of GSI, where some part of this work has been done. J.C. gratefully acknowledges the hospitality of the physics department of the University of Bielefeld. This work was supported in part by BMBF under the program of scientific-technological collaboration (WTZ project RUS-656-96).

References

[1] For a review see e.g. J. Sollfrank, J. of Physics G **23**, (1997) 1903 and references therein.

[2] J. Cleymans and K. Redlich, Phys. Rev.Lett. **81** (1998) 5284.

[3] J. Cleymans and K. Redlich, nucl-th/9903063.

[4] C.M. Hung and E.V. Shuryak, Phys. Rev. Lett. **75** (1995) 4003; Phys. Rev. C **57** (1998) 1891.

[5] E.G. Nikonov, A.A. Shanenko and V.D. Toneev, Heavy Ion Physics **8** (1998) 89; nucl-th/9802018.

[6] V.D. Toneev, E.G. Nikonov and A.A. Shanenko, Proceedings of Interdisciplinary Workshop on Nuclear Matter in Different Phases and Transitions, Les Houches, France, March 30 - April 10, 1998, Kluwer Academic Publ. in Fundamental Theories of Physics (to be published); Preprint GSI 98-30, Darmstadt, 1998.

[7] See e.g. F. Karsch, Talk given at the "Strong and Electroweak Matter '98", December 1998, Copenhagen, hep-lat/9903031.

[8] Zimanyi J. *et al.* Nucl. Phys.A**484** (1988) 647.

[9] J. Cleymans, R.V. Gavai and E. Suhonen, Phys. Rep. **130** (1986) 217.

[10] See e.g. D.H. Rischke, in "Hadrons in Dense Matter and Hadrosynthesis", pp. 21-70, Ed. J. Cleymans, H.B. Geyer and F.G. Scholtz, Springer, (1999).

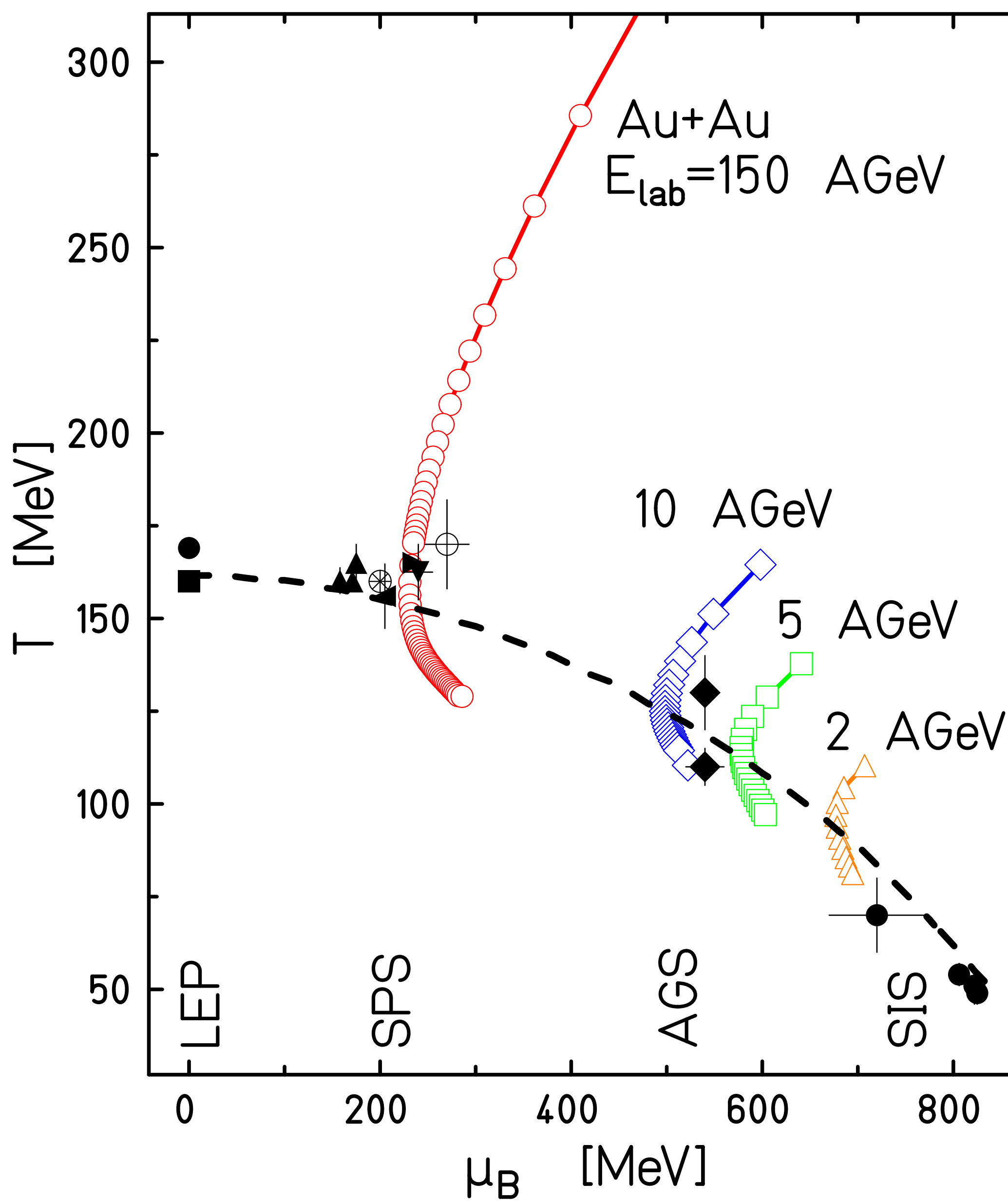
[11] J. Cleymans, H. Oeschler and K. Redlich, Phys. Rev. C **59** (1999) 1663.

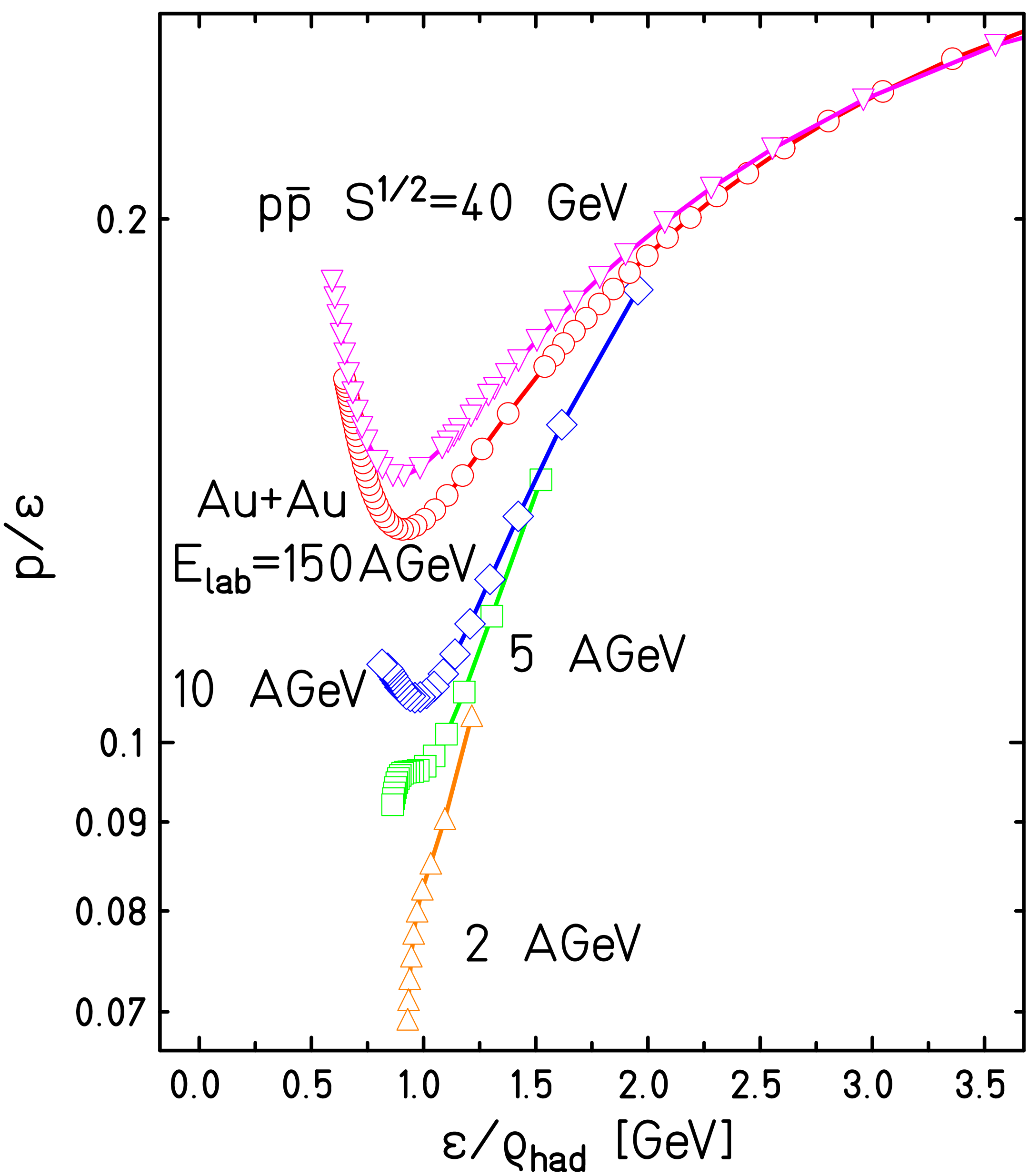
[12] L.D. Landau, Izv. Akad. Nauk SSSR, **17** (1953) 51.

Figure captions

Fig.1. The compiled chemical freeze-out parameters (borrowed from [2, 3]) obtained from the observed hadronic abundances and dynamical trajectories calculated for central $Au+Au$ collisions at different beam energies E_{lab} with the mixed phase equation of state. The smooth dashed curve is calculated in the hadronic gas model for $\langle E_{had} \rangle / \langle N_{had} \rangle = 1\ GeV$ [2].

Fig.2. The ratio of pressure to energy density, p/ε, versus the average energy per hadron, ε/ρ_{had}, for evolution of different systems. The upper curve corresponds to $p\bar{p}$ collisions at $\sqrt{S} = 40\ GeV$ with isoentropic expansion from a sphere with $R = 1\ fm$. Other cases are calculated for central $Au + Au$ collisions at the given beam energy under the same conditions as in Fig.1.


Au+Au
E_{lab}=150 AGeV
10 AGeV
5 AGeV
2 AGeV
LEP
SPS
AGS
SIS
T [MeV]
μ_B [MeV]
300
250
200
150
100
50
0
200
400
600
800



p/ε
ε/ϱhad [GeV]
pp̄ S1/2=40 GeV
Au+Au
Elab=150 AGeV
10 AGeV
5 AGeV
2 AGeV
0.07
0.08
0.09
0.1
0.2
0.0
0.5
1.0
1.5
2.0
2.5
3.0
3.5